Notes on Dual Labor Markets and Labor Protection in an Estimated Search and Matching Model

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- The use of temporary contracts is extensive in European countries and they also have proliferated in developing countries in the last 20 years.
- *Duality:* Two types of contracts with different durations and termination costs: Permanent vs. Temporary.
- *Theory:* Effect on productivity growth: (1) Employer-paid training (Carpio et.al. 2011) and (2) Workers' effort (Dolado and Stucchi, 2008).
- *Policy:* Are temporary contracts used to reintroduce some flexibility when firing costs are high? (Cahuc and Postel-Vinay, 2002; Meghir et. al., 2012).
  - Temporary contracts increase flexibility.
  - Firing costs are aimed to reduce unemployment.
- A large part of the literature has treated the use of temporary contracts as exogenous. Also, the emphasis has been primarily on European countries.

This paper:

- Estimates a structural search model using supply side data for an Emerging Economy (Chile) with dual labor markets in which the use of temporary contracts is endogenous.
- Quantitatively evaluate the role of the labor protection legislation and the use of temporary contracts on welfare and inequality.

Related literature: This paper is related with various streams of literature:

- *Firing Costs and Temporary Contracts:*
  - *Exogenous determination of Temporary contracts:* Bentolila and Saint-Paul (1992), Bentolila and Dolado (1994), Saint-Paul (1996), Wasmer (1999), Garibaldi and Violante (1999), Goux and Maurin (2000), Blanchard and Landier (2000), **Cahuc and Postel-Vinay (2002)**.
  - *Endogenous determination of Temporary contracts:* ***Cao et. al. (2011)***, *Alvarez and Veracierto (2012), Macho-Stadler et.al. (2012), Paolini and Tena (2012).*
- *Estimation of equilibrium search models:* Flinn and Heckman (1982), Eckstein and Wolpin (1990), Postel-Vinay and Robin (2002), **Flinn (2006)**.

- Search and matching model extended to allow for temporary and permanent contracts.
- Distinction between contracts:
  - Pay-roll Taxes.
  - Severance Payments.
  - Productivity Gains.
- Four states: Unemployed, a new employee with a permanent contract (OP), a continuing employee with a permanent contract (IP), and employed with a temporary contract (T).
- Unemployed workers receive a flow utility b and there is no on-the-job search.
- Time is continuos and individuals are ex-ante homogeneous.
- Labor market policy:
  - Contract specific pay-roll taxes: τ_i and ϕ_i (for $i = P, T$) for workers and employers.
  - Severance tax: Ψ. The tax receipts are thrown into the ocean.
- Firms search for workers and pay the search cost k_i (for $i = P, T$).

The Model

Workers' Value functions

Unemployed:

$$rU = b + \alpha_w^P \int_{x_{OP}^*}^{\infty} \{W_{OP}(x) - U\} \, dF_P(x) + \alpha_w^T \int_{x_T^*}^{\infty} \{W_T(x) - U\} \, dF_T(x)$$

Employed as an **outsider** with a **permanent contract**:

$$\begin{aligned} rW_{OP}(x) \quad &= \quad w_{OP}(x)\,(1-\tau_P) + \lambda_P \int_{x_{IP}^*}^{\infty} W_{IP}(x') f_P(x') dx' + \lambda_P F_P(x_{IP}^*) U \\ &\quad -\lambda_P W_{OP}(x) \end{aligned}$$

Employed as an **insider** with a **permanent contract**:

$$\begin{aligned} rW_{IP}(x) \quad &= \quad w_{IP}(x)\,(1-\tau_P) + \lambda_P \int_{x_{IP}^*}^{\infty} W_{IP}(x') f_P(x') dx' + \lambda_P F_P(x_{IP}^*) U \\ &\quad -\lambda_P W_{IP}(x) \end{aligned}$$

Employed with a **temporary contract**:

$$rW_T(x) = w_T(x)\,(1-\tau_T) + \lambda_T U - \lambda_T W_T(x)$$

The Model

Firms' Value functions

Filled vacancy with an **outsider** with **permanent contract**:

$$\begin{aligned} rJ_{OP}(x) &= x - w_{OP}(x)(1+\phi_P) + \lambda_P F_P(x^*_{IP})\left(V_P - J_{OP}(x) - \Psi\right) \\ &\quad + \lambda_P \int_{x^*_{IP}}^{\infty} \left\{J_{IP}(x') - J_{OP}(x)]\right\} f_P(x')dx' \end{aligned}$$

Filled vacancy with an **insider** with **permanent contract**:

$$\begin{aligned} rJ_{IP}(x) &= x - w_{IP}(x)(1+\phi_P) + \lambda_P F_P(x^*_{IP})\left(V_P - J_{IP}(x) - \Psi\right) \\ &\quad + \lambda_P \int_{x^*_{IP}}^{\infty} \left\{J_{IP}(x') - J_{IP}(x)\right\} f_P(x')dx' \end{aligned}$$

Filled vacancy with **temporary contract**:

$$rJ_T(x) = x - w_T(x)(1+\phi_T) + \lambda_T\left(V_T - J_T(x)\right)$$

Unfilled vacancy:

$$\begin{aligned} rV_P &= -k_P + \alpha_e^P \int_{x^*_{OP}}^{\infty} \left\{J_{OP}(x) - V_P\right\} dF_P(x) \\ rV_T &= -k_T + \alpha_e^T \int_{x^*_{T}}^{\infty} \left\{J_T(x) - V_T\right\} dF_T(x) \end{aligned}$$

Wage are determined using Nash Bargaining. The maximization problems are:

$$\max_{\{w_{OP}(x)\}} (W_{OP}(x) - U)^{\beta_P} (J_{OP}(x) - V_P)^{1-\beta_P}$$

$$\max_{\{w_{IP}(x)\}} (W_{IP}(x) - U)^{\beta_P} (J_{IP}(x) - V_P + \Psi)^{1-\beta_P}$$

$$\max_{\{w_T(x)\}} (W_T(x) - U)^{\beta_T} (J_T(x) - V_T)^{1-\beta_T}$$

Wage equations:

$$w_{OP}(x) = \frac{\beta_P (x - \lambda_P \Psi) + (1-\beta_P)\frac{(1+\phi_P)}{(1-\tau_P)} rU}{(1+\phi_P)}$$

$$w_{IP}(x) = \frac{\beta_P (x + r\Psi) + (1-\beta_P)\frac{(1+\phi_P)}{(1-\tau_P)} rU}{(1+\phi_P)}$$

$$w_T(x) = \frac{\beta_T x + (1-\beta_T)\frac{(1+\phi_T)}{(1-\tau_T)} rU}{(1+\phi_T)}$$

Note that: $w_{IP}(x) > w_{OP}(x)$

Definition (Steady-State Equilibrium)

Given a vector of parameters $(b, \lambda_P, \lambda_T, r, \beta_P, \beta_T, k_P, k_T)$, a matching function $m(\cdot)$, a vector of taxes $(\tau_P, \tau_T, \phi_P, \phi_T, \Psi)$, and a probability distribution function for productivity for each type of contract $F_P(x)$ and $F_T(x)$, a steady-state equilibrium in a dual labor market economy is a labor market tightness q and a proportion of vacancies of jobs with permanent contracts η, together with reservation productivities x_i^* for $i = OP, IP, T$, unemployment rate u and employment rates e_P and e_T such that:

1. Given q and η (and rU), the reservation productivities x_i^* for $i = OP, IP, T$ solve:

$$\begin{aligned} x_{IP}^* &= \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi - \frac{\lambda_P}{r+\lambda_P} \int_{x_{IP}^*}^{\infty} \left(x' - x_{IP}^*\right) f_P(x') dx' \\ x_{OP}^* &= x_{IP}^* + (\lambda_P + r)\,\Psi \\ x_T^* &= \frac{(1+\phi_T)}{(1-\tau_T)} rU \end{aligned}$$

where $rU = b + \left(\frac{1-\tau_P}{1+\phi_P}\right) \frac{\eta q \beta_P k_P}{(1-\beta_P)} + \left(\frac{1-\tau_T}{1+\phi_T}\right) \frac{(1-\eta) q \beta_T k_T}{(1-\beta_T)}$.

Definition (Steady-State Equilibrium, Cont...)

2. q and η solve the following system in equations and are consisted with the reservation productivities x_i^* for $i = OP, IP, T$.

$$k_P = \frac{m\left[\eta q\right]}{\eta q} \frac{(1-\beta_P)}{(r+\lambda_P)} \int_{x_{OP}^*}^{\infty} (x - x_{OP}^*)\, dF_P(x)$$
$$k_T = \frac{m\left[(1-\eta)\, q\right]}{(1-\eta)\, q} \frac{(1-\beta_T)}{(r+\lambda_T)} \int_{x_T^*}^{\infty} (x - x_T^*)\, dF_T(x)$$

3. Given the reservation productivities x_i^* for $i = OP, IP, T$, the unemployment rate u and employment rates e_P and e_T satisfy:

$$\alpha_w^P \left[1 - F_P(x_{OP}^*)\right] u = \lambda_P F_P(x_{IP}^*) e_P$$
$$\alpha_w^T \left[1 - F_T(x_T^*)\right] u = \lambda_T e_T$$

where $e_P + e_T + u = 1$:

Return

Data

Chile: *"Encuesta de Protección Social"*

- *Description:* Panel data with labor histories.
- *Estimation Sample:* A cross-section data is constructed including just spells that contain December 2006. Information about transition from unemployment to both type of jobs is necessary.
- *Data Available:* Unemployment duration, hourly wages and durations in both types of jobs, and transitions from unemployment to both types of jobs.

$$\{t_u, w_P, t_{e_P}, w_T, t_{e_T}, I(u \rightarrow e_P), I(u \rightarrow e_T)\}$$

- *Sample Restrictions and Homogeneity Controls*:
  - Sample: male, head of household, without college, and aged between 25 and 60 years.
  - Outliers: 2.5% on the top and bottom of wages were dropped.
- *Pay-roll taxes and employment protection:* $\tau_P = 0.206$, $\phi_P = 0.016$, $\tau_T = 0$, $\phi_T = 0.03$, and $\Psi = 12\bar{w_P}$. See details

Data

Descriptive Statistics

	Mean	S.D.
Wages (Dollars per Hour)		
$w \mid e_P$	2.69	2.21
$w \mid e_T$	1.65	0.85
Ratio	1.63	
Duration (Months)		
$t \mid u$	16.57	12.28
% Left Censored	6.14	
% Right Censored	15.79	
$t \mid e_P$	105.93	81.78
% Left Censored	2.12	
% Right Censored	81.52	
$t \mid e_T$	26.01	23.89
% Left Censored	1.54	
% Right Censored	61.4	
Transitions (Percent)		
$u \rightarrow e_P$	22.8	
$u \rightarrow e_T$	61.4	
Share by Type of Contract (Percent)		
Permanent	77.92	
Temporary	22.08	

Sample: Men, head of household, between 25 and 60 years old, and without college degree.

Unemployment duration contribution:

- Bover and Gomez (2004): Distinguish between exits to a permanent job and to a temporary job.
- Define the following intensities of transition to each of the states:

$$h_u^P = \alpha_w^P \left[1 - F(x_{OP}^*)\right] \text{ and } h_u^T = \alpha_w^T \left[1 - F(x_T^*)\right]$$

- The likelihood contribution of unemployment duration is:

$$f_u(t_{i,u}, i \in U) = \left[h_u^P \exp(-h_u^P t_u)\right]^{D_P} \left[h_u^T \exp(-h_u^T t_u)\right]^{D_T} u \quad t_u > 0$$

- For the case of censored spells:

$$\begin{aligned} f_u(t_{i,u}, i &\in U, c_i^l = 1) = \Pr[T \leq t_u] = \left[1 - \exp(-h_u t_u)\right] u \quad t_u > 0 \\ f_u(t_{i,u}, i &\in U, c_i^r = 1) = \Pr[T > t_u] = \exp(-h_u t_u) u \quad t_u > 0 \end{aligned}$$

Wages contribution:

- Wages contribution for the permanent contracts (Note: conditional on the model, $\Pr(OP) = \Pr[\text{receive } 0 \text{ shocks in } t] = \exp(-\lambda_P t_{e_P})$):

$$g(w_i, i \in E_P | w_i > w_P(x^*{}_P), P) = \left[\frac{\exp\left(-\lambda_P t_{i,e_P}\right) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU + \lambda_P \Psi \right)}{1 - G(w_P(x^*{}_P)|P, OP)} + \frac{(1 - \exp\left(-\lambda_P t_{i,e_P}\right)) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi \right)}{1 - G(w_P(x^*{}_P)|P, IP)} \right] e_P$$

- Wages contribution for temporary contracts:

$$g(w_i, i \in E_T | w_i > w_T(x_T^*), T) = \frac{\frac{(1+\phi_T)}{\beta_T} f_T \left(w_i \frac{(1+\phi_T)}{\beta_T} - \frac{(1-\beta_T)}{\beta_T} \frac{(1+\phi_T)}{(1-\tau_T)} rU \right)}{1 - G(w_T(x_T^*)|T)} e_T$$

The likelihood function:

$$
\begin{aligned}
L(\Theta^{SS}; w, t) \quad = \quad & \prod_{i=1}^{N} \left[f_u(t_{i,u}, i \in U)\right]^{u \times (1-c_i^l) \times (1-c_i^r)} \times \left[f_u(t_{i,u}, i \in U, c_i^l = 1)\right]^{u \times c_i^l \times (1-c_i^r)} \\
& \times \left[f_u(t_{i,u}, i \in U, c_i^r = 1)\right]^{u \times (1-c_i^l) \times c_i^r} \\
& \times \left[\int_{w_P(x_P^*)} \frac{1}{w_i} m\left(\frac{w_i^o}{w_i}\right) g(w_i, i \in E_P | w_i > w_P(x_P^*), P) dw_i\right]^{e_P \times (1-u)} \\
& \times \left[\int_{w_T(x_T^*)} \frac{1}{w_i} m\left(\frac{w_i^o}{w_i}\right) g(w_i, i \in E_T | w_i > w_T(x_T^*), T) dw_i\right]^{(1-e_p) \times (1-u)}
\end{aligned}
$$

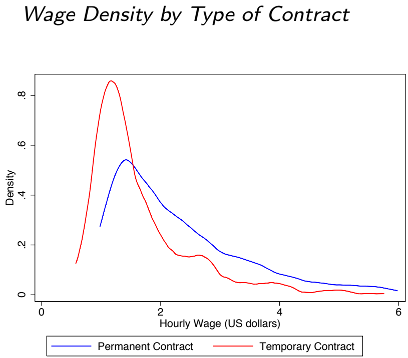
This likelihood is maximized subject to the following equilibrium constraints:

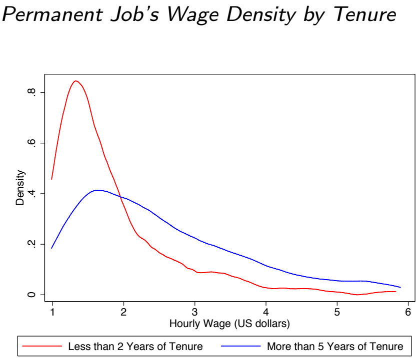
$$
\begin{aligned}
x_{IP}^* &= \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi - \frac{\lambda_P}{r+\lambda_P} \int_{x_{IP}^*}^{\infty} \left(x' - x_{IP}^*\right) f_P(x') dx' \\
x_{OP}^* &= x_{IP}^* + (\lambda_P + r)\,\Psi \\
x_T^* &= \frac{(1+\phi_T)}{(1-\tau_T)} rU
\end{aligned}
$$

choosing: $\Theta^{SS} = \left\{\alpha_w^P, \alpha_w^T, \lambda_P, \lambda_T, x_T^*, x_{IP}^*, x_{OP}^*, rU, \mu_x^P, \sigma_x^P, \mu_x^T, \sigma_x^T, \sigma_\epsilon\right\}$

Identification with Supply Side Information: Flinn and Heckman (1982), Flinn (2006). *Supply side parameters:*

- $\Theta^{SS} = \{\alpha_w^P, \alpha_w^T, \lambda_P, \lambda_T, rU, x^*_{OP}, x^*_{IP}, x^*_T, \mu_x^P, \sigma_x^P, \mu_x^T, \sigma_x^T\}$ are identified from the LF.
    - Recoverability condition and location-scale family: Log-normality is assumed.
    - α_w^P and α_w^T are identified from the transition from unemployment to both types of jobs.
    - λ_P and λ_T are identified from steady state equilibrium conditions.
    - Differences in wages distributions (between types of jobs and with different tenures) identifies productivity distributions.
- The Nash bargaining coefficient β is difficult to identify without demand side information (Eckstein and Wolpin, 1995). Assume: $\beta_P = \beta_T = \beta = 0.5$.
- The discount factor can not be identified. Assume: $r = 0.0053$ (Fuenzalida and Mongrut, 2010).


Wage Density by Type of Contract
Density
0
.2
.4
.6
.8
0
2
4
6
Hourly Wage (US dollars)
Permanent Contract
Temporary Contract


Permanent Job's Wage Density by Tenure
Density
0
.2
.4
.6
.8
1
2
3
4
5
6
Hourly Wage (US dollars)
Less than 2 Years of Tenure
More than 5 Years of Tenure

Demand side parameters:

- $\Theta^{DS} = \{k_P, k_T, \eta, q, m(\cdot)\}$ are identified under particular assumptions.
  - Without information on v, additional parameters in $m(\cdot)$ cannot be identified.
  - Knowledge of $m(\cdot)$ is sufficient to identify the cost of vacancies k_P, k_T.
    - Assume that $m(\cdot)$ contains no unknown parameters: $m(u,v) = v(1 - exp(-u/v))$
    - External estimates of a Cobb-Douglas matching function parameter
- Identification of the parameters $q, \eta, m(\cdot), k_P, k_T$ build on the estimators of Θ^{SS}:
  - η and q solve

$$\widehat{\alpha}_w^P = m\left[\eta q\right]$$
$$\widehat{\alpha}_w^T = m\left[(1-\eta)\, q\right]$$

    given $m(\cdot)$ identified
  - k_P, k_T are recovered from the free entry conditions:

$$k_P = \frac{m\left[\widehat{\eta}\widehat{q}\right]}{\widehat{\eta}\widehat{q}} \frac{(1-\beta_P)}{(r+\widehat{\lambda}_P)} \int_{\widehat{x}^*_{OP}}^{\infty} \left(x - \widehat{x}^*_{OP}\right) dF_P(x)$$
$$k_T = \frac{m\left[(1-\widehat{\eta})\, \widehat{q}\right]}{(1-\widehat{\eta})\, \widehat{q}} \frac{(1-\beta_T)}{(r+\widehat{\lambda}_T)} \int_{\widehat{x}^*_{T}}^{\infty} \left(x - \widehat{x}^*_{T}\right) dF_T(x)$$

Preference Parameter:

- b can the be recovered from value function of unemployment.

$$b = \widehat{rU} - \left(\frac{1-\tau_P}{1+\phi_P}\right)\frac{\widehat{\eta}\widehat{q}\beta_P\widehat{k}_P}{(1-\beta_P)} - \left(\frac{1-\tau_T}{1+\phi_T}\right)\frac{(1-\widehat{\eta})\,\widehat{q}\beta_T\widehat{k}_T}{(1-\beta_T)}$$

Estimation Results

Estimation Results

Estimated Parameters

	Param.	Std.Err.(*)
α_w^P	0.1362	0.0004
α_w^T	0.3475	0.0022
λ_P	0.0015	0.000001
λ_T	0.0127	0.00004
x_T^*	1.1256	0.0459
x_{IP}^*	1.1211	0.0598
x_{OP}^*	1.3422	0.0598
η	0.2493	0.0008
q	0.3841	0.0025
k_P	45.3040	1.0640
k_T	6.2272	0.4860
b	-4.0413	0.2838
No. Obs.	2,170	
Loglik	-4,859	
F-test $\mu_P^x = \mu_T^x$, $\sigma_P^x = \sigma_T^x$	231	

(*) Asymptotic standard errors.

Note: Technological parameters estimated using a Cobb Douglas Matching Function ($m(q) = q^\gamma$, $\gamma = 0.85$)

▸ Predicted Values

Counterfactual and Policy Experiments

Description

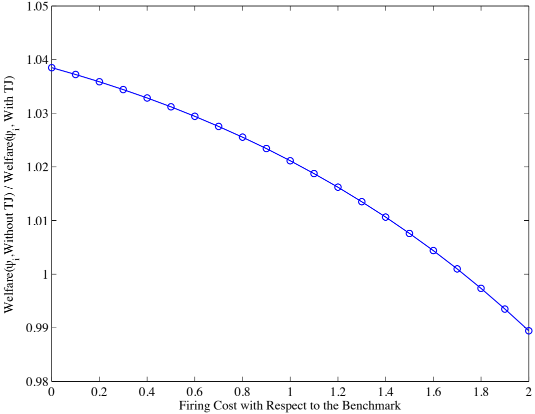
- **Counterfactual Experiment:** What is the effect of temporary jobs?
  - Benchmark economy vs. an economy without temporary contracts.
  - Effect on labor market dynamics and wages.
- **Policy Experiment:** What is the effect of changes in firing costs?
  - I analyzed a range from zero firing cost to twice the benchmark.
  - Benchmark economy vs. an economy without temporary contracts.
  - Effect on labor market dynamics and wages.
- **Welfare Analysis:**
  - Compare changes in welfare when firing costs change in both cases, with and without temporary contracts.
- Counterfactual and policy experiments uses a Cobb-Douglas matching function with $\gamma = 0.85$.

Counterfactual and Policy Experiments

Labor Market Dynamics, Productivity and Inequality

	TC Allowed			TC Not Allowed		
	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$
Market Tightness						
q	0.363	0.384	0.405	0.142	0.141	0.141
η	0.277	0.249	0.225	1.000	1.000	1.000
Arrival Rates						
α_w^P	0.142	0.136	0.130	0.191	0.190	0.189
α_w^T	0.321	0.347	0.373	n.a.	n.a.	n.a.
Labor Market Status						
u	0.049	0.049	0.048	0.040	0.038	0.036
e_P	0.763	0.743	0.724	0.960	0.962	0.964
e_T	0.189	0.209	0.227	n.a.	n.a.	n.a.
Reservation Productivity						
x^*_{OP}	1.320	1.342	1.365	1.168	1.171	1.173
x^*_{IP}	1.320	1.121	0.923	1.168	0.949	0.731
x^*_T	1.142	1.126	1.111	n.a.	n.a.	n.a.
$Temporary/Outsider$	0.601	0.596	0.592	n.a.	n.a.	n.a.

Effect of Changes in Firing Cost: Temporary Jobs Allowed and not Allowed



▸ See definitions

Conclusions

Estimation Results

- Under the estimated parameters, both types of contracts survive in equilibrium.
- Temporary jobs arrive more often than permanent jobs (2.6 times faster), but permanent workers are, on average, 30% more productive (wage differences are 60%, on average).
- Only 25% of the available vacancies are related with permanent contracts.

Conterfactual and Policy Experiments Results

- Labor protection is useful to reduce unemployment. Temporary contracts cancels out this effect.
- Labor protection increases the (equilibrium) employment rate of jobs with temporary contracts. Strong substitution is observed.

Policy Implications

- Temporary contracts increase flexibility but they do not make workers and firms better off (productivity gains effect).
- Stringent labor protection generates important trade offs between workers' and firms' welfare.

Data

- *Pay-roll taxes and employment protection:*

	Permanent Contract		Temporary Contract	
	Worker	Employer	Worker	Employer
Pay-roll Taxes	$\tau_P = 0.206$	$\phi_P = 0.016$	$\tau_T = 0$	$\phi_T = 0.03$
Retirement	0.1	0	0	0
Health	0.07	0	0	0
Disability	0.03	0	0	0
Unemployment Insurance	0.006	0.016	0.03	0
Severance Tax	n.a.	$\Psi = 12\bar{w_P}$	n.a.	0

Sources: Edwards and Cox NBER (2000), Fajnzylber et.al. 2009, and World Bank (2006)

Return

Parameter Identification in MLE (Formal Proof)

To formally show identification I am going to use Flabbi (2010) strategy. The likelihood function was:

$$
\begin{aligned}
L(\Theta^{SS}; w, t) \quad = \quad & \prod_{i=1}^{N} \left[f_u(t_{i,u}, i \in U) \right]^u \\
& \times \left[g(w_i, i \in E_P | w_i > w_P(x_P^*), P) \right]^{e_P \times (1-u)} \\
& \times \left[g(w_i, i \in E_T | w_i > w_T(x_T^*), T) \right]^{(1-e_p) \times (1-u)}
\end{aligned}
$$

Alternatively:

$$
\begin{aligned}
\ln L(\Theta^{SS}; w, t) \quad = \quad & \sum_{i \in N_u} \ln f_u(t_{i,u}, i \in U) + \sum_{i \in N_P} \ln g(w_i, i \in E_P | w_i > w_P(x_P^*), P) \\
& + \sum_{i \in N_T} \ln g(w_i, i \in E_T | w_i > w_T(x_T^*), T)
\end{aligned}
$$

Parameter Identification in MLE (Formal Proof)

Therefore:

$$
\begin{aligned}
\ln L(\Theta^{SS}; w, t) \quad = \quad & \sum_{i \in N_{u,P}} \ln \left[h_u^P \exp(-h_u^P t_{u,i}) \right] + \sum_{i \in N_{u,T}} \ln \left[h_u^T \exp(-h_u^T t_{u,i}) \right] + \sum_{i \in N_u} \ln u \\
& \sum_{i \in N_T} \ln \left[\frac{\frac{(1+\phi_T)}{\beta_T} f_T \left(w_i \frac{(1+\phi_T)}{\beta_T} - \frac{(1-\beta_T)}{\beta_T} \frac{(1+\phi_T)}{(1-\tau_T)} rU \right)}{1 - F_T(x_T^*)} \right] + \sum_{i \in N_T} \ln e_T \\
& \sum_{i \in N_P} \ln \left[\frac{\exp(-\lambda_P t_{i,e_P}) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU + \lambda_P \Psi \right)}{1 - F_P(x_{OP}^*)} \right. \\
& \left. + \frac{(1 - \exp(-\lambda_P t_{i,e_P})) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi \right)}{1 - F_P(x_{IP}^*)} \right] \\
& + \sum_{i \in N_P} \ln e_P
\end{aligned}
$$

Parameter Identification in MLE (Formal Proof)

Lets first consider the contribution of unemployment duration data:

$$N_{u,P}h_u^P - h_u^P \sum_{i \in N_{u,P}} t_{u,i} + N_{u,T}h_u^T - h_u^T \sum_{i \in N_{u,T}} t_{u,i} + N_u \ln u + N_T \ln e_T + N_P \ln e_P$$

Recall:

$$u = \frac{h_E^T h_E^P}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P}, \; e_P = \frac{h_u^P h_E^T}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P}, \; e_T = \frac{h_u^T h_E^P}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P}$$

where:

$$h_u^P = \alpha_w^P \left[1 - F_P(x_{OP}^*)\right], \; h_u^T = \alpha_w^T \left[1 - F_T(x_T^*)\right], \; h_E^P = \lambda_P F_P(x_{IP}^*), \; h_E^T = \lambda_T$$

Therefore:

$$N_{u,P}h_u^P - h_u^P \sum_{i \in N_{u,P}} t_{u,i} + N_{u,T}h_u^T - h_u^T \sum_{i \in N_{u,T}} t_{u,i} + (N_u + N_T) \ln h_E^P + (N_u + N_P) h_E^T$$

$$N_T \ln h_u^T + N_P \ln h_u^P + N \ln \left(h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P\right)$$

Parameter Identification in MLE (Formal Proof)

FOC:

$$h_u^P: \quad N_{u,P} - \sum_{i \in N_{u,P}} t_{u,i} + N_P \frac{1}{h_u^P} + N \frac{1}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P} h_E^T = 0$$

$$h_u^T: \quad N_{u,T} - \sum_{i \in N_{u,T}} t_{u,i} + N_T \frac{1}{h_u^T} + N \frac{1}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P} h_E^P = 0$$

$$h_E^P: \quad (N_u + N_T) \frac{1}{h_E^P} + N \frac{1}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P} \left(h_u^T + h_E^T \right) = 0$$

$$h_E^T: \quad (N_u + N_P) \frac{1}{h_E^T} + N \frac{1}{h_u^P h_E^T + h_u^T h_E^P + h_E^T h_E^P} \left(h_u^P + h_E^P \right) = 0$$

The system can be solved for the four unknowns. So the hazard rates out of unemployment and employment are identified just with unemployment duration data and the transitions between unemployment to both types of contracts.

In terms of the model parameters:

$$h_u^P = \alpha_w^P \left[1 - F_P(x_{OP}^*) \right] \quad (1)$$

$$h_u^T = \alpha_w^T \left[1 - F_T(x_T^*) \right] \quad (2)$$

$$h_E^P = \lambda_P F_P(x_{IP}^*) \quad (3)$$

$$h_E^T = \lambda_T \quad (4)$$

Parameter Identification in MLE (Formal Proof)

The contribution to the likelihood of wage of temporary workers was:

$$\sum_{i \in N_T} \ln \left[\frac{\frac{(1+\phi_T)}{\beta_T} f_T \left(w_i \frac{(1+\phi_T)}{\beta_T} - \frac{(1-\beta_T)}{\beta_T} \frac{(1+\phi_T)}{(1-\tau_T)} rU \right)}{1 - F_T(x_T^*)} \right]$$

Now, using location and scale parameters we have:

$$\sum_{i \in N_T} \ln \left[\frac{\frac{1}{S_T} f_T \left(\frac{w_i - L_T}{S_T} \right)}{1 - F_T(x_T^*)} \right]$$

where:

$$L_T = \frac{(1-\beta_T)}{(1-\tau_T)} rU + \frac{\beta_T}{(1+\phi_T)} \mu_T^x \qquad (5)$$

$$S_T = \frac{\beta_T}{(1+\phi_T)} \sigma_T^x \qquad (6)$$

L_T and S_T are identified from temporary jobs wage data.

Parameter Identification in MLE (Formal Proof)

The contribution to the likelihood of wage of permanent workers was:

$$\sum_{i \in N_P} \ln \left[\frac{\exp(-\lambda_P t_{i,e_P}) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU + \lambda_P \Psi \right)}{1 - F_P(x^*_{OP})} \right.$$
$$\left. + \frac{(1 - \exp(-\lambda_P t_{i,e_P})) \frac{(1+\phi_P)}{\beta_P} f_P \left(w_i \frac{(1+\phi_P)}{\beta_P} - \frac{(1-\beta_P)}{\beta_P} \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi \right)}{1 - F_P(x^*_{IP})} \right]$$

Now, using location and scale parameters we have:

$$\sum_{i \in N_P} \ln \left[\frac{\exp(-\lambda_P t_{i,e_P}) \frac{1}{S_{OP}} f_P \left(\frac{w_i - L_{OP}}{S_{OP}} \right)}{1 - F_P(x^*_{OP})} + \frac{(1 - \exp(-\lambda_P t_{i,e_P})) \frac{1}{S_{IP}} f_P \left(\frac{w_i - L_{IP}}{S_{IP}} \right)}{1 - F_P(x^*_{IP})} \right]$$

Parameter Identification in MLE (Formal Proof)

where:

$$L_{OP} = \frac{(1-\beta_P)}{(1-\tau_P)} rU + \lambda_P \Psi + \frac{\beta_P}{(1+\phi_P)} \mu_P^x \tag{7}$$

$$L_{IP} = \frac{(1-\beta_P)}{(1-\tau_P)} rU - r\Psi + \frac{\beta_P}{(1+\phi_P)} \mu_P^x \tag{8}$$

$$S_{OP} = S_{IP} = S_P = \frac{\beta_P}{(1+\phi_P)} \sigma_P^x \tag{9}$$

The contribution of permanent job wages is a mixture of two truncated normal distributions that share the same scale parameter. Because the weights change in a deterministic way, Teicher (1963) result apply so $(L_{OP}, L_{IP}, S_P, \lambda_P)$ are identified from wage data. Finally the model restrictions are:

$$x^*_{OP} = x^*_{IP} + (\lambda_P + r)\,\Psi \tag{10}$$

$$x^*_T = \frac{(1+\phi_T)}{(1-\tau_T)} rU \tag{11}$$

$$x^*_{IP} = \frac{(1+\phi_P)}{(1-\tau_P)} rU - r\Psi - \lambda_P \frac{(1+\phi_P)}{(1-\tau_P)} \int_{x^*_{IP}}^{\infty} S_{IP}(x') f_P(x') dx' \tag{12}$$

Solving equations (13) to (24) for twelve unknowns I can recover all the parameters. Return

Estimation Results

Predicted Values

	Value	Std.Err. (*)	Data
	Productivity		
$E(x_P)$	0.885	0.00206	
$V(x_P)$	9.957	0.10310	
$E(x_T)$	0.678	0.01159	
$V(x_T)$	1.060	0.05803	
	Offered Wages		
$E(w_{OP})$	1.106	0.02703	
$E(w_{IP})$	1.217	0.02702	
$E(w_T)$	0.886	0.01648	
	Accepted Wages		
$E(w_{OP}\|e_P)$	2.857	0.08946	2.69
$E(w_{IP}\|e_P)$	2.719	0.09172	2.69
$E(w_T\|e_T)$	1.704	0.04459	1.65
	Labor Market Status		
u	0.049	0.00291	0.05
e_P	0.743	0.00206	0.74
e_T	0.209	0.00497	0.21
	Labor Market Dynamics		
h_u	0.074	0.00563	0.060
h_{e_P}	0.001	0.00001	0.009
h_{e_T}	0.013	0.00004	0.038

(*) Standard Errors calculated using delta method.

Return

Labor Market Dynamics

	TC Allowed			TC Not Allowed		
	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$
Market Tightness						
q	0.363	0.384	0.405	0.142	0.141	0.141
η	0.277	0.249	0.225	1.000	1.000	1.000
Arrival Rates						
α_w^P	0.142	0.136	0.130	0.191	0.190	0.189
α_w^T	0.321	0.347	0.373	n.a.	n.a.	n.a.
Labor Market Status						
u	0.049	0.049	0.048	0.040	0.038	0.036
e_P	0.763	0.743	0.724	0.960	0.962	0.964
e_T	0.189	0.209	0.227	n.a.	n.a.	n.a.
Duration						
h_U	0.0698	0.0738	0.0777	0.0312	0.0309	0.0307
h_{e_P}	0.0013	0.0013	0.0012	0.0013	0.0012	0.0012
h_{e_T}	0.0127	0.0127	0.0127	n.a.	n.a.	n.a.

Counterfactual and Policy Experiments

Productivity and Wages

Reservation Productivity and Wages

	TC Allowed			TC Not Allowed		
	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$	$0 \times \Psi$	$1 \times \Psi$	$2 \times \Psi$
Reservation Productivity						
x^*_{OP}	1.320	1.342	1.365	1.168	1.171	1.173
x^*_{IP}	1.320	1.121	0.923	1.168	0.949	0.731
x^*_{T}	1.142	1.126	1.111	n.a.	n.a.	n.a.
Accepted Wages						
$E(w_{OP}\|e_P)$	2.867	2.857	2.848	2.625	2.584	2.544
$E(w_{IP}\|e_P)$	2.867	2.719	2.567	2.625	2.441	2.248
$E(w_{T}\|e_T)$	1.724	1.703	1.685	n.a.	n.a.	n.a.
Inequality						
$Outsider/Insider$	1.000	1.051	1.109	1.000	1.059	1.132
$Temporary/Outsider$	0.601	0.596	0.592	n.a.	n.a.	n.a.
$Temporary/Insider$	0.601	0.626	0.656	n.a.	n.a.	n.a.

- Flinn (2006) and Flabbi (2010): Long run welfare impact of changes in policy.
  - At any point in time agents are unemployed, employed under permanent contract or employed under temporary contract.
  - At any point in time firms with permanent/temporary contracts have filled or unfilled vacancies. Unfilled vacancies has by definition value zero (free entry condition).
  - The Social Welfare function is:

$$\begin{aligned} S(\tau, \phi, \Psi) \quad = \quad & u(\tau, \phi, \Psi) U_u(\tau, \phi, \Psi) + e_{OP}(\tau, \phi, \Psi) \left[\bar{W}_{OP}(\tau, \phi, \Psi) + \bar{J}_{OP}(\tau, \phi, \Psi) \right] \\ & + e_{IP}(\tau, \phi, \Psi) \left[\bar{W}_{IP}(\tau, \phi, \Psi) + \bar{J}_{IP}(\tau, \phi, \Psi) \right] \\ & + e_T \left[\bar{W}_T(\tau, \phi, \Psi) + \bar{J}_T(\tau, \phi, \Psi) \right] \end{aligned}$$

where: $\tau = (\tau_P, \tau_T)$, $\phi = (\phi_P, \phi_T)$, $U_u(\tau, \phi, \Psi)$ is unemployed agents welfare, $\bar{V}_j(\tau, \phi, \Psi)$ is the average workers welfare and $\bar{J}_j(\tau, \phi, \Psi)$ is the average firms welfare ($j = OP, IP, T$).

Welfare Analysis

Description

- Define the following:

$$
\begin{aligned}
U_u(\tau,\phi,\Psi) &= \int_0^{\min\{x^*_{IP},x^*_T\}} U\left[\frac{f_P(x)}{F_P(x^*_{IP})} I_{[x^*_{IP}\leq x^*_T]} + \frac{f_T(x)}{F_T(x^*_T)}\left(1 - I_{[x^*_{IP}\leq x^*_T]}\right)\right] \\
\bar{V}_j(\tau,\phi,\Psi) &= \int_{x^*_j}^{\infty} W_j(x)\left[\frac{f_P(x)}{1-F_P(x^*_j)}\right] dx \quad j = IP, OP \\
\bar{V}_T(\tau,\phi,\Psi) &= \int_{x^*_T}^{\infty} W_T(x)\left[\frac{f_T(x)}{1-F_T(x^*_T)}\right] dx \\
\bar{J}_j(\tau,\phi,\Psi) &= \int_{x^*_j}^{\infty} J_j(x)\left[\frac{f_P(x)}{1-F_P(x^*_j)}\right] dx \quad j = IP, OP \\
\bar{J}_T(\tau,\phi,\Psi) &= \int_{x^*_T}^{\infty} J_T(x))\left[\frac{f_T(x)}{1-F_T(x^*_T)}\right] dx
\end{aligned}
$$

Return